

CSC 302 1.5 Neural Networks

Tutorial

Problem #1

The input to a single-input to the network is 2.0, its weight is 2.3 and its bias is -3.

- i) What is the net input to the transfer function?
- ii) What is the neuron output?
- iii) What is the output of the neuron if it has the following transfer function?
 - a. Hard limit
 - b. Linear
 - c. Log-Sigmoid

Problem #2

Given a two-input neuron with the following parameters $b = 1.2$, $W = [3 \ 2]$ and $p = [-5 \ 6]^T$, calculate the neuron output for the following transfer functions?

- i) A symmetric hard limit transfer function
- ii) A saturating linear transfer function
- iii) A hyperbolic tangent sigmoid (tansig) transfer function

Problem #3

A single-layer neural network is to have six inputs and two outputs. The outputs are to be limited to and continuous over the range 0 to 1. What can you tell about the network architecture?

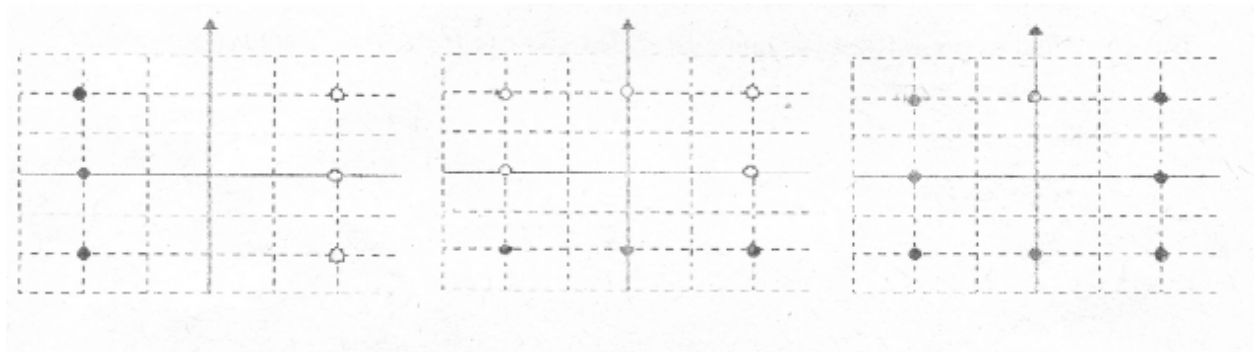
Specifically

- i) How many neurons are required?
- ii) What are the dimensions of the weight matrix?

- iii) What kind of transfer function could be used?
- iv) Is a bias required?

Problem #4

Solve the three simple classification problems shown below by drawing a decision boundary. Find weights and bias values that result in single-neuron perceptrons with the chosen decision boundaries.



Problem #5

We have a classification problem with four classes of input vectors. The four classes are

$$\text{Class 1 : } \left\{ p_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, p_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$$

$$\text{Class 2 : } \left\{ p_3 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, p_4 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right\}$$

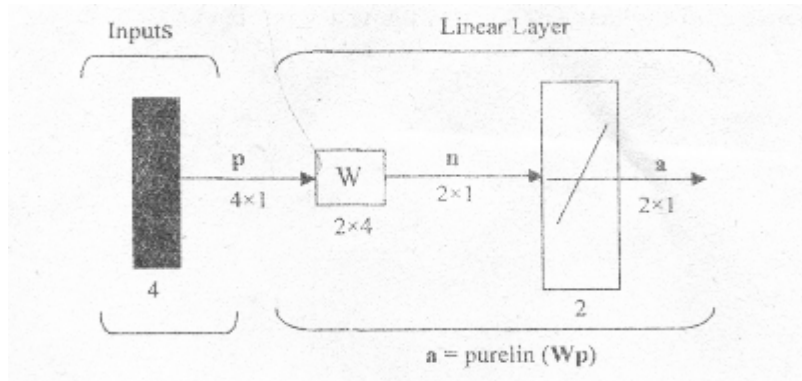
$$\text{Class 3 : } \left\{ p_5 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, p_6 = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$$

$$\text{Class 4 : } \left\{ p_7 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, p_8 = \begin{bmatrix} -2 \\ -2 \end{bmatrix} \right\}$$

- i) Design a perceptron network to solve this problem.
- ii) Train the network to solve their problem using the perceptron learning rule.

Problem #6

Consider the linear associate shown below.



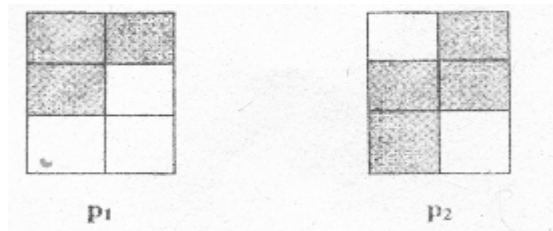
Let the input/output prototype vectors be

$$\left\{ p_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, t_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\} \left\{ p_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, t_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

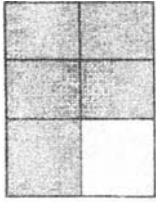
- i) Use the Hebb rule to find the appropriate weight matrix for this linear associator.
- ii) Repeat part (i) using pseudo inverse rule.
- iii) Apply the input p_1 to linear associator using the weight matrix of part (i), then using the weight matrix of part (ii).

Problem #7

Consider the prototype patterns shown below.



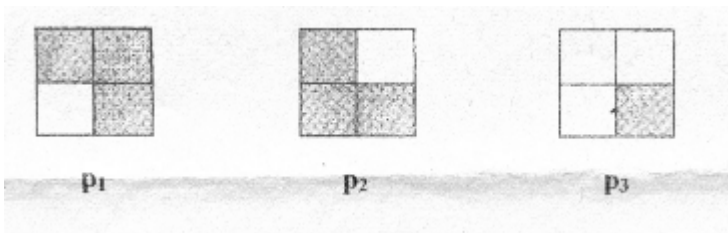
- i) Are these patterns orthogonal?
- ii) Design an autoassociator for these patterns. Use the Hebb rule.
- iii) What response does the network give to the test input pattern, p_t shown below?



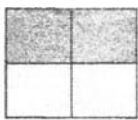
P_t

Problem #8

Consider the following prototype patterns.



- i) Use the Hebb rule to design a perceptron network that will recognize these three patterns.
- ii) Find the response of the network to the pattern **PI** given below. Is the response correct?

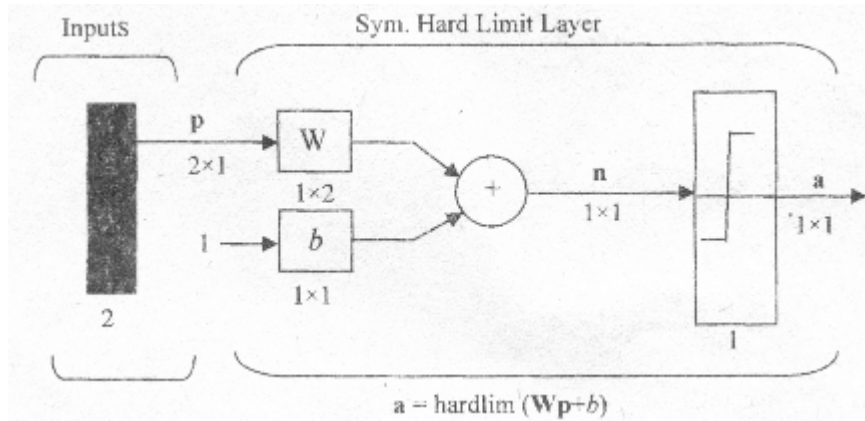


P_t

Problem #9

Consider the problem of designing a perceptron network (see figure) to recognize the following patterns.

$$p_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad p_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$



- i) Why is a bias required to solve this problem?
- ii) Use the pseudo inverse rule to design a network with bias to solve this problem.

Problem #10

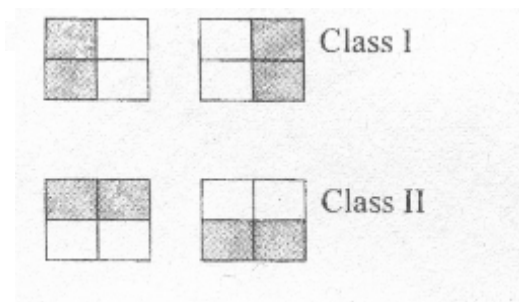
Suppose that we have the following input/target pairs:

$$\left\{ p_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, t_1 = 1 \right\} \quad \left\{ p_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, t_2 = -1 \right\}$$

These patterns occur with equal probability, and they are used to train an ADALINE network with no bias. Train the network using the LMS algorithm, with the initial guess set to zero and a learning rate $\alpha = 0.25$. Apply each reference pattern only once during training. Draw the decision boundary at each stage.

Problem #11

Consider the two classes of patterns that are shown in the following figure. Class I represents vertical lines and class II represents horizontal lines.



- i) Are these categories linearly separable?
- ii) Design a multilayer network to distinguish these categories.

Problem #12.

Show that a multilayer network with linear transfer functions is equivalent to a single-layer linear network.

Problem #13

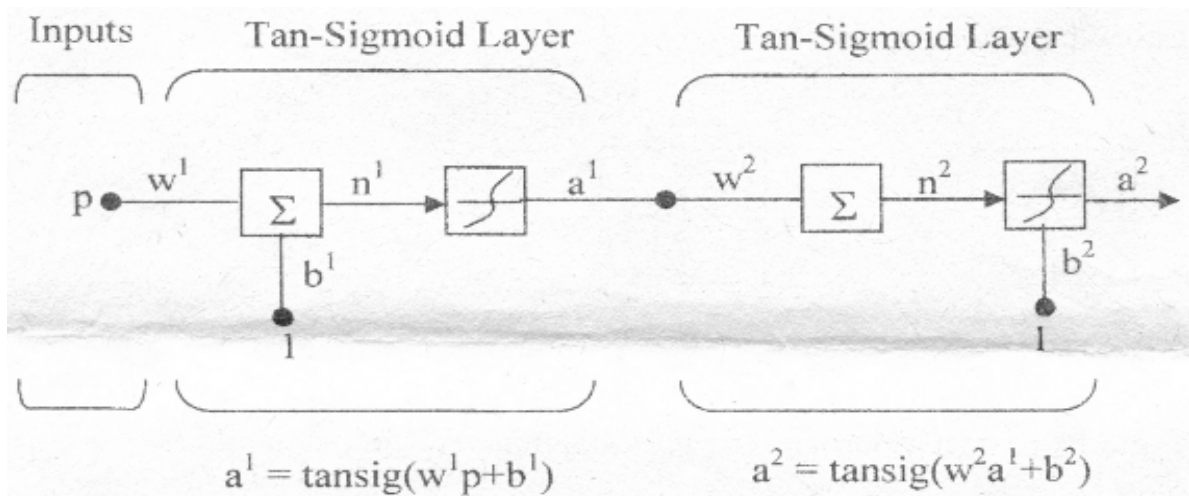
For the network shown below, the initial weights and biases are chosen to be

$$w^1(0) = -1, b^1(0) = 1, w^2(0) = -2, b^2(0) = 1$$

An input/target pair is given to be

$$P = -1, t = 1$$

Perform one iteration of back propagation with $\hat{d} = 1$.



Problem #14

Show that backpropagation reduces to the LMS algorithm for a single-layer linear network (ADALINE).